THE APPLICATION OF OWAs IN EXPERTISE PROCESSES: THE DEVELOPMENT OF A MODEL FOR THE QUANTIFICATION OF HIDDEN QUALITY COSTS

Abstract. This paper will introduce a fuzzy model for the quantification of hidden quality costs based on the aggregation of subjective information. The proposed model will be able to properly aggregate and summarize subjective opinions expressed by experts about the costs to be quantified, thereby achieving an adequate level of objectivity. To do so, a Probabilistic Uncertain Ordered Weighted Average operator is used, establishing as weighting factors both the confidence the organization has in each expert and, thanks to an original and specifically designed tool, the company’s position on Crosby’s well-known Quality Management Maturity Grid. Finally, in order to refine the results, the values obtained will undergo Contra-Expertise through Ordered Weighted Average. Once the theoretical model has been described, it will be applied to a case study: the quantification of the cost of loss of image in one insurance brokerage firm.

Keywords: Quality Management; Quality Cost; Fuzzy Logic; Ordered Weighted Average; Case Study.

JEL Classification: C02; M10;M49

1. Introduction
The first written reference about the term “quality cost” was by Juran at the beginning of the 1950s. Specifically, his book Quality Control Handbook (Juran, 1951) defines what he calls the “cost of poor quality” as “the sum of all costs that would disappear if there were no quality problems.” This position relates very closely to Campanella’s point of view, according to which: “any cost that would not have occurred if the quality were perfect, contributes to quality cost” (Campanella, 1999).

The essence of measuring quality cost can be found in the fact that each identified quality problem brings with it a visible recoverable cost which can be assigned a value (Campanella, 1999). However, what happens when, because of
the nature of the problem, it is impossible to carry out an objective valuation of the quality cost?

The aim of this paper is to answer this question. A new methodology will be developed, which we believe will improve the quantification process of certain elements of quality cost, which are called “hidden” because of the inherent difficulties for their estimation.

A great part of the literature written about the measurement of hidden quality costs stems from the study by Kotler (1991), and in particular, that by Albright and Roth (1992), where different methods for calculating such costs are outlined. Since then, several authors have dealt with the quantification of hidden quality costs from different perspectives (see for example, Robison, 1997; Chiadamrong, 2003; Freiesleben, 2004; Snieska et al., 2013). But authors have traditionally used Probabilistic Theory as a mathematical instrument to quantify hidden quality costs. However, the use of Probabilistic Theory requires assigning precise numbers to each event, when in reality, as the estimations they are, they would be better described by vague assertions, and therefore, approximate. Indeed, such theory does not contemplate the imprecision and subjectivity underlying any process of quantifying hidden quality costs. In order to overcome this problem, we propose using fuzzy logic and the concept of possibility.

The application of fuzzy set theory is a suitable approach in cases where uncertainty is due to incompleteness or imprecision. Several authors have dealt with uncertainty using fuzzy sets (e.g., Klir and Yuan, 1995; Zimmermann, 2000; Zadeh, 2005; Brotons and Terceño, 2010). On the other hand, applying fuzzy logic in management accounting is not new. For example Terceño and Vigier (2011) developed a model of financial economic diagnosis of companies, which on the basis of a simple scheme of cause-effect, simulate the action of the analyst in its task of diagnosis.

This paper will introduce a model for the quantification of hidden quality costs based on the aggregation of subjective information. In order to improve the treatment of the subjectivity and uncertainty existing in information supplied by experts, the use of the Experton Theory (Kaufmann, 1987) and application of OWAs are proposed. This is a very interesting instrument which permits the aggregation of the experts’ opinion and gives consistency to the results. Since Yager (1988) introduced this aggregation technique based on ordered weighted averaging (OWA), extensive literature has been published about it (Sadiq and Tesfamariam, 2007; Zarghami and Szidarovszky, 2009; Dong et al., 2010; Casanovas et al., 2015). However, this paper introduces a completely innovative application of the tools described for their use in the management and measurement of quality. Added to this, the model has an original development towards the aggregation of experts’ opinions. These opinions are weighted according to the level of confidence in each expert and the company’s position on Crosby’s Quality Management Maturity Grid (Crosby, 1979), which should be especially noted for its originality.
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2. Preliminaries

Definition 1: An ordered weighted average (OWA) is defined as a mapping of dimension n, $F : \mathbb{R}^n \rightarrow \mathbb{R}$ that has an associated weighting vector $W$ of dimension $n$, $W^T = [w_1, w_2, ..., w_n]$, such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, with

$$f(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j \cdot b_j$$  \hspace{1cm} (1)

Where $b_j$ is the jth largest of the $a_i$.

The essence of OWA (Yager, 1988) is the rearrangement of the elements or arguments, causing aggregation in the $a_j$ not associated with a weighting $w_j$ but with the placement order instead.

Definition 2: A BADD-OWA operator (Yager, 1993; Yager and Filev, 1994) is defined as a mapping of dimension n, $F : \mathbb{R}^n \rightarrow \mathbb{R}$ that has an associated weighting vector $W$ of dimension $n$, $W^T = [w_1, w_2, ..., w_n]$ that meets the following condition $w_j = b^\gamma_j / \sum_{j=1}^{n} b^\gamma_j$, with

$$f(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j \cdot b_j$$  \hspace{1cm} (2)

Where $\alpha \in (-\infty, +\infty)$, $b_j$ is the jth largest of the $a_i$.

Definition 3: An uncertain OWA operator (UOWA) (Xu and Da, 2002) is defined as a mapping of dimension n, $F : \Omega^n \rightarrow \Omega$ that has an associated weighting vector $W$ of dimension $n$, $W^T = [w_1, w_2, ..., w_n]$, such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, with

$$\text{UOWA}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} w_j \cdot b_j^*$$  \hspace{1cm} (3)
Where $b_j^*$ is the jth largest of the $\tilde{a}_i$. The $\tilde{a}_i$ ($i \in N$) are trust intervals and are defined either as simple triplets.

**Definition 4.** A probabilistic uncertain OWA operator (UPOWA) (Merigó and Wei, 2011) is defined as a mapping of dimension $n$, $F: \Omega^n \rightarrow \Omega$ that has an associated weighting vector $W$ of dimension $n$, $W^T = [w_1, w_2, \ldots, w_n]$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$ a vector of probabilities $V = (v_1, v_2, \ldots, v_n)^T$ such that $v_j \in [0, 1]$ and $\sum_{j=1}^{n} v_j = 1$ where,

$$UPOWA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \beta \cdot \sum_{j=1}^{n} w_j \cdot b_j^* + (1 - \beta) \sum_{j=1}^{n} v_j \cdot a_i$$  \hspace{1cm} (4)

Where $\alpha \in (-\infty, +\infty)$, $b_j^*$ is the jth largest of the $\tilde{a}_i$, $\tilde{a}_i$ are confidence intervals defined as simple triplets.

**Definition 5.** BADD probabilistic uncertain OWA operator (BADD-UPOWA) is defined as a mapping of dimension $n$, $F: \Omega^n \rightarrow \Omega$ that has an associated weighting vector $W$ of dimension $n$, $W^T = [w_1, w_2, \ldots, w_n]$ such that

$w_j = b_j^{\alpha} / \sum_{j=1}^{n} b_j^{\alpha}$, and a vector of probabilities $V = (v_1, v_2, \ldots, v_n)^T$ such that $v_j \in [0, 1]$ and $\sum_{j=1}^{n} v_j = 1$ where,

$$BADD-UPOWA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \beta \cdot \sum_{j=1}^{n} w_j \cdot b_j^* + (1 - \beta) \sum_{j=1}^{n} v_j \cdot a_i$$  \hspace{1cm} (5)

Where $b_j^*$ is the jth largest of the $\tilde{a}_i$, $\tilde{a}_i$ are confidence intervals defines either as simple triplets.

3. **Methodology**

The sections below outline in detail the different stages of an innovative model that enables quantification of any cost that the company considers a hidden quality cost.
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3.1. First valuation of Hidden Quality Cost by H experts

First of all, based on their knowledge and the information provided a priori by the organization, a number of H experts are asked to carry out a first valuation of the Hidden Quality Cost to be quantified. Since the experts are likely to use approximations, the information will be gathered through confidence triplets or triangular fuzzy numbers (TFN).

\[ \bar{Q}_i = (q_{i1}^1, q_{i2}^2, q_{i3}^3), \forall i = 1, \ldots, H \] (6)

The results given by the different experts are aggregated and summarized by applying BADD-UPOWA. To do so, the experts’ opinions are weighted according to the level of confidence the company has in each of them and the company’s position on Crosby’s Quality Management Maturity Grid, which is particularly innovative.

3.2. Use of the Quality Management Maturity Grid for the aggregation of the experts' predictions about Hidden Quality Cost.

Crosby is an important reference in the study of the behavior of Quality Costs in organizations. Through his Quality Management Maturity Grid he analyzed the evolution of quality costs with respect to the development of Quality Management by simply observing the attitude of the human component of the organization towards it.

The grid created by Crosby identifies five stages of maturity which describe the different phases a company goes through and the consequent costs incurred. These stages go from ignorance and total lack of confidence in quality to reaching an ideal situation where quality management is considered an essential part of the organization.

According to Crosby, although the companies situated at the first stage “Uncertainty” do not make any estimation of Quality Cost, these costs can reach and even surpass 20% of sales. As a company promotes the implementation of quality and advances through the stages of the Maturity Grid, Quality Cost decreases. Therefore, at the “Awakening” stage these costs are around 18%, at the “Enlightenment” stage they reach approximately 12% of sales, at the “Wisdom” stage they reach 8% and finally at the “Certainty” stage Quality Cost can represent 2.5% of sales (Crosby, 1979).

Whether we more or less agree or disagree with the percentages provided by this prestigious author, the fact is that as a company advances in Crosby's Maturity Grid by suitably reinforcing quality management, it will make fewer mistakes and these mistakes will also incur lower costs. Therefore, Quality Cost in general and hidden quality cost in particular will decrease.

On the basis of this premise, the company’s position on Crosby's Maturity Grid will be introduced as a weighting factor. This will be done in such a way that for companies situated at the first stages of the table, the highest values given by
the H experts will be weighted more, and in contrast, for the companies situated in the final stages, more importance will be given to the lower values.

The development of this innovative proposal requires positioning the company on Crosby’s Maturity Grid, so it will be necessary to have the opinion of new experts, logically different from the previous experts, who could be internal (with the necessary training) or external to the company.

3.2.1 Determining the company’s membership for each of the stages of Crosby’s Maturity Grid.

A group of L experts will assess the company’s membership for each of Crosby’s proposed stages: Uncertainty (A_1), Awakening (A_2), Enlightenment (A_3), Wisdom (A_4) and Certainty (A_5). The experts will use a scale of six elements, 1 (totally disagree), 2 (strongly disagree), 3 (disagree), 4 (neutral), 5 (true) and 6 (very true). The respondent’s position for each proposition, which is uncertain, is considered a fuzzy subset, and the six possible values the respondent may take is what we will call referential. Thus, we can speak of a level of membership \( \mu_j \), \( j = 1, ..., 6 \). The membership function assigned to each of the previous labels is shown in Table 1.

<table>
<thead>
<tr>
<th>Linguistic label</th>
<th>( \mu_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Totally disagree</td>
<td>0.00</td>
</tr>
<tr>
<td>2 Strongly disagree</td>
<td>0.20</td>
</tr>
<tr>
<td>3 Disagree</td>
<td>0.40</td>
</tr>
<tr>
<td>4 Neutral</td>
<td>0.60</td>
</tr>
<tr>
<td>5 True</td>
<td>0.80</td>
</tr>
<tr>
<td>6 Very true</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In short, this attempts to overcome the problems of measuring the different alternatives for each situation. The results made available by the experts for each group are summarized in Table 2, where \( a_{ij} \) indicates the number of experts that value stage i with the j grade on the previous scale of six elements.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>I_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>a_{11}</td>
<td>a_{12}</td>
<td>a_{13}</td>
<td>a_{14}</td>
<td>a_{15}</td>
<td>a_{16}</td>
<td>I_1</td>
</tr>
<tr>
<td>A_2</td>
<td>a_{21}</td>
<td>a_{22}</td>
<td>a_{23}</td>
<td>a_{24}</td>
<td>a_{25}</td>
<td>a_{26}</td>
<td>I_2</td>
</tr>
<tr>
<td>A_3</td>
<td>a_{31}</td>
<td>a_{32}</td>
<td>a_{33}</td>
<td>a_{34}</td>
<td>a_{35}</td>
<td>a_{36}</td>
<td>I_3</td>
</tr>
<tr>
<td>A_4</td>
<td>a_{41}</td>
<td>a_{42}</td>
<td>a_{43}</td>
<td>a_{44}</td>
<td>a_{45}</td>
<td>a_{46}</td>
<td>I_4</td>
</tr>
<tr>
<td>A_5</td>
<td>a_{51}</td>
<td>a_{52}</td>
<td>a_{53}</td>
<td>a_{54}</td>
<td>a_{55}</td>
<td>a_{56}</td>
<td>I_5</td>
</tr>
</tbody>
</table>

For its part, the stage j, index is obtained as
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\[ I_j = \frac{1}{L} \sum_{k=1}^{5} \mu_i a_{ik}, j = 1, 2, \ldots, 5. \]  

(7)

Where \( L \) is the number of experts that value the company’s membership function for each of the stages of Crosby's Maturity Grid.

The total stage index \( I_T \) is obtained as:

\[ I_T = \sum_{j=1}^{5} j \cdot I_j / \sum_{j=1}^{5} I_j \]  

(8)

The stage the company is situated at on Crosby's Maturity Grid is determined through the total Stage index \( I_t \). However, it is now necessary to define to what degree exactly it belongs to each one.

Degree of membership to Uncertainty stage (A₁),

\[ \mu_1 (I_T) = \begin{cases} 2 - I_T & 1 < I_T < 2 \\ 0 & \text{otherwise} \end{cases} \]  

(9)

Degree of membership to Awakening stage (A₂),

\[ \mu_2 (I_T) = \begin{cases} I_t - 1 & 1 < I_T < 2 \\ 3 - I_t & 2 < I_T < 3 \\ 0 & \text{otherwise} \end{cases} \]  

(10)

Degree of membership to Enlightenment stage (A₃),

\[ \mu_3 (I_T) = \begin{cases} 4 - I_t & 3 < I_T < 4 \\ 0 & \text{otherwise} \end{cases} \]  

(11)

Degree of membership to Wisdom stage (A₄),

\[ \mu_4 (I_T) = \begin{cases} I_T - 3 & 3 < I_T < 4 \\ 5 - I_T & 4 < I_T < 5 \\ 0 & \text{otherwise} \end{cases} \]  

(12)

Degree of membership to Certainty stage (A₅),

\[ \mu_5 (I_T) = \begin{cases} I_T - 4 & 4 < I_T < 5 \\ 0 & \text{otherwise} \end{cases} \]  

(13)

Through the previously outlined definitions of the stages, the result will indicate the company’s simultaneous membership to two of them with their corresponding membership functions. In any case, the total of all the membership functions will be equal to 1.
3.2.2. Determination of the values of hidden quality cost at each stage.

The costs given by the H experts are aggregated in section 1 by applying BADD-OWA, with different weightings for each of the TFN extremes, and by applying a different $\gamma$ for each stage of Crosby's Maturity Grid (Table 3).

Table 3. Assignation of $\gamma$ coefficient to each stage of Crosby's Maturity Grid

<table>
<thead>
<tr>
<th>Stage</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty ($A_1$)</td>
<td>2</td>
</tr>
<tr>
<td>Awakening ($A_2$)</td>
<td>1</td>
</tr>
<tr>
<td>Enlightenment ($A_3$)</td>
<td>0</td>
</tr>
<tr>
<td>Wisdom ($A_4$)</td>
<td>-1</td>
</tr>
<tr>
<td>Certainty ($A_5$)</td>
<td>-2</td>
</tr>
</tbody>
</table>

In this way, the experts who give higher values of hidden quality cost are weighted much more if the company is at stage $A_1$, a little more if it is at stage $A_2$, all equally at stage $A_3$, a little more for those who provide lower values at phase $A_4$ and a lot more for those who provide lower values at phase $A_5$.

Different weightings are used for each of the extremes of TFN hidden quality cost provided by the experts (6). That is to say, the weightings of the central values and those of the lower and higher extremes differ from each other.

\[
\omega_{S_j}^r = q_{l,j}^{r,j-3} / \sum_{l=1}^{m} q_{l,j}^{r,j-3}, r = 1, 2, 3 \text{ and } j = 1, \ldots, 5 \quad (14)
\]

Where $\omega_{S_j}^r$ is the weighting of extreme $r$ (1, lower; 2, central and 3, higher) of the stage of Crosby's Maturity Grid ($j$), and $l$ indicates $l$-th expert with the greatest value communicated. Based on these weightings the TFN hidden quality cost is obtained for stage $j$. $Q_{S_j} = (Q_{S_j}^1, Q_{S_j}^2, \ldots, Q_{S_j}^5)$, $j = 1, 2, \ldots, 5$, where,

\[
Q_{S_j}^r = \sum_{l=1}^{m} q_{l,j}^{r,j-2} / \sum_{l=1}^{m} q_{l,j}^{r,j-3}, r = 1, 2, 3 \text{ and } j = 1, \ldots, 5 \quad (15)
\]

$j = 1, \ldots, 5$ corresponding to the phases $A_1, \ldots, A_5$. In this way a different hidden quality cost is obtained for each of the stages of Crosby's Maturity Grid the company belongs to.

3.2.3. Hidden quality cost value taking the company's position on Crosby's Maturity Grid as the weighting factor.
Multiplying the costs obtained in each phase by the company’s membership function will be enough to determine the costs the company has incurred. The costs of each stage are determined by expression (15) and the membership function to each phase is determined by expressions (9) to (13). The result is the TFN hidden quality cost weighted according to the company’s position on Crosby’s Maturity Grid \( \tilde{Q}_C = (Q_{C_1}, Q_{C_2}, Q_{C_3}) \), being

\[
Q_{C_i}^r = \sum_{j=1}^{5} \mu_j (I_T) \cdot Q_{S_j}^r, \ r = 1, 2, 3
\]  

(16)

3.3. Value of hidden quality cost incorporating the importance attributed to each expert as weighting factor

Although the hidden quality cost obtained according to expression (16) weights the initial valuations of the \( H \) experts (6) in relation to the company’s position on Crosby’s Maturity Grid (expressions (9) to (13)), it does not take into account the level of confidence the company believes the \( H \) experts merit.

Through BADD-UPOWA, it is possible to obtain the cost value by considering the company’s position on Crosby’s Maturity Grid and incorporating the confidence in each of the selected \( H \) experts. The aggregation of the two weighting factors considered is reflected in expression (17) whose result is a first approximation to the final hidden quality cost \( \tilde{Q}_F = (Q_{F_1}, Q_{F_2}, Q_{F_3}) \)

\[
\tilde{Q}_F = \beta \cdot \tilde{Q}_C + (1 - \beta) \sum_{i=1}^{H} v_i \cdot \tilde{Q}_i
\]  

(17)

Where,

\( \beta \): takes values between 0 and 1 and indicates the importance assigned to the company’s position on Crosby’s Maturity Grid as weighting factor

\( 1 - \beta \): indicates the importance assigned to the degree of confidence in the \( H \) experts as weighting factor, regardless of the values they expressed.

\( v_i \): probability assigned to the expert \( i \) according to degree of confidence the company has in the expert.

As can be seen, through \( \beta \) each organization will decide which of the two weighting factors, an existing culture of quality or confidence in the experts, will have a greater specific weight in the final valuation.
Although $\tilde{Q}_F = \{Q^1_F, Q^2_F, Q^3_F\}$ summarizes the final hidden quality cost value analyzed, in order to provide a greater consistency to the analysis carried out, we propose that the results obtained undergo Contra-Expertise.

### 3.4. Revision of the information obtained applying the BADD-UPOWA Contra-Expertise method: final quantification of hidden quality cost.

Contra-Expertise consists of applying to new experts, different from those who made the first expertise, who will comment on the opinions of the first experts, thereby increasing the objectivity of the new opinions about the resulting values. The application of BADD-UPOWA in the grouping of the new opinions about hidden quality cost, permits introducing the same weighting factors as before into the analysis. The following three stages are proposed:

### 3.4.1. Calculation of hidden quality cost according to the company’s position on Crosby’s Maturity Grid

First we obtain the M-Expertons, defined as Experton, where opinions are provided in triplets of confidence instead of in confidence intervals or true values, which permits greater sensibility as well as greater approximation by the experts. Afterwards, the M-Expertons, defined in the interval $[0,1]$, will be transformed into R-Expertons, whose interval comprehends the universe of values that the variable can take, in this case hidden quality cost expressed in Euros. The steps to be followed in this first stage are given below:

a) A new group of $H$ experts have to indicate their level of conformity with the values obtained in (17). It is recommended that the same number of participants as in the valuation made by the first experts be maintained in order to avoid weightings being distorted. Should they not coincide in number, it would be necessary to once again obtain the weightings of expression (14) in section 3.2.2. By giving their opinion about a TFN, the experts will facilitate a triplet of values $(t^i_1, t^i_2, t^i_3), i' = 1, \ldots, H$, which we will denominate level of conformity. 11 $\gamma$ levels will be considered, from 0 to 1 so that if they consider that the Hidden Quality Cost is $Q_1$, they will value it with 0, and if they consider that it is $Q_3$, they will value it with 1 (Table 4).

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Hidden quality cost (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Is correct $Q^1_F$</td>
</tr>
<tr>
<td>0.1</td>
<td>Practically $Q^1_F$</td>
</tr>
<tr>
<td>0.2</td>
<td>Almost $Q^1_F$</td>
</tr>
<tr>
<td>0.3</td>
<td>Approaching $Q^1_F$</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>More approaching to $Q_F^1$ than $Q_F^3$</td>
</tr>
<tr>
<td>0.5</td>
<td>Approaching as $Q_F^1$ as $Q_F^3$</td>
</tr>
<tr>
<td>0.6</td>
<td>More approaching to $Q_F^3$ than $Q_F^3$</td>
</tr>
<tr>
<td>0.7</td>
<td>Approaching a $Q_F^3$</td>
</tr>
<tr>
<td>0.8</td>
<td>Almost $Q_F^3$</td>
</tr>
<tr>
<td>0.9</td>
<td>Practically $Q_F^3$</td>
</tr>
<tr>
<td>1.0</td>
<td>Is correct $Q_F^3$</td>
</tr>
</tbody>
</table>

b) The previous opinions are in descending order and assigned the weightings obtained in section 3.2.2 according to (14). So, $\omega_{Si'}^r$ corresponds to the weighting from the $r$ extreme of the TFN, of the element that occupies position $i'$ in descending order. In this way, a set of pairs of values is obtained $(t^r_i, \omega_{Si'}^r)$, where the first makes reference to the expert's opinion and the second to the weight that corresponds to it.

c) Obtaining relative frequencies of each element of the triplet for the 11 $\gamma$ levels, as well as for each one of the five stages of Crosby's Maturity Grid.

$$fr^r_{\gamma,i,j} = \sum_{i' = 1}^{H} \omega_{Si'}^r / t^r_i = \gamma, r = 1, 2, 3, j = 1, \ldots, 5$$

(18)

d) Calculation of the accumulated frequencies ($fa^r_{\gamma, j}$) according to the weights assigned to the previous relative frequencies (from $\gamma = 0$ to $\gamma = 1$). This will also be repeated for each of the five stages of Crosby's Maturity Grid, therefore, an M-experton will have been constructed for each of them.

$$fa^r_{\gamma, j} = \sum_{\gamma = \gamma^*}^{H} fr^r_{\gamma, j}, r = 1, 2, 3, j = 1, \ldots, 5$$

(19)

In this way the weightings assigned to all the experts in each of the 11 levels are added.

e) Obtaining the expected value of the M-expertons. For each of the M-expertons defined for each of the stages of Crosby's Maturity Grid the TFN $T_j = (T_j^1, T_j^2, T_j^3)$ is obtained as

$$T_j^r = \frac{1}{10} \sum_{\gamma = 0.1}^{1} fa^r_{\gamma, j}, r = 1, 2, 3, j = 1, \ldots, 5$$

(20)

That is to say all levels are considered except $\gamma = 0$. 83
f) The triplets obtained for each of the stages according to expression (20) are multiplied by the degree of membership to these stages, expressions (9) to (13), thereby obtaining the triplet \( L = (L_1, L_2, L_3) \), where

\[
L_i^r = \sum_{j=1}^{5} \mu_j (I_{T}) \cdot T_j^r, \quad r = 1, 2, 3
\]  

(21)

In this way, based on the five triplets (one per stage), a triplet summarizing the information provided by the experts is achieved.

g) Calculation of the R-Experton hidden quality cost. Its value \( \tilde{R} = (R_1, R_2, R_3) \), is obtained as,

\[
R_i^r = Q_i^r + \left( Q_i^3 - Q_i^1 \right) \cdot L_i^r, \quad r = 1, 2, 3
\]  

(22)

However, the result obtained does not take into consideration the company's confidence in each of the experts that participated in the Contra-Expertise.

3.4.2. Obtaining hidden quality cost according to the confidence inspired by the experts

a) Assignation of probabilities \( \tilde{u}_{i^*} \), \( i^* = 1, \ldots, H \) to each of the experts participating in the Contra-Expertise in order to weight their opinions according to the level of confidence they deserve from the company and obtaining the corresponding M-experton \( \tilde{P} = (P^1, P^2, P^3) \), being,

\[
P_i^r = \sum_{i=1}^{1} t_{i^*} \cdot \tilde{u}_{i^*}, \quad r = 1, 2, 3
\]  

(23)

b) Obtaining the R-experton corresponding to the confidence inspired by the experts \( \tilde{S} = (S^1, S^2, S^3) \),

\[
S_i^r = Q_i^r + \left( Q_i^3 - Q_i^1 \right) \cdot P_i^r, \quad r = 1, 2, 3
\]  

(24)

3.4.3. Final valuation of hidden quality cost

Aggregation of the costs by taking into account the weighting factors considered \( i \) is reflected in expression (25), where the company will define the weight that it wants to give each of these factors through a coefficient \( \tilde{\beta}^* \in [0,1] \). In this way, it is possible to summarize the final value of the hidden quality cost through the TFN \( \tilde{Q}_{IF} = (Q_{IF}^1, Q_{IF}^2, Q_{IF}^3) \).
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\[ Q_{FT} = \beta^r \cdot R^r + (1-\beta^r)S^r \]  \hspace{1cm} (25)

Although the triplet obtained informs about the values that hidden quality cost ranges between, if required, this TFN could be defuzzified through any of the known methods obtaining the final result through a crisp value.

4. Case study

After developing the theoretical model, we decided to apply the proposed methodology to a real case. For this purpose, we collaborated with an insurance brokerage firm with an insurance portfolio for 2015 valued at 1.2 million euros, which can be considered representative of the industry average in Spain. Hidden quality cost par excellence, as much for its importance as well as the extreme difficulty in measuring it, is the loss of income as a consequence of deterioration in the image of the company (Freiesleben, 2004). In this sense, it was decided to quantify the cost incurred from the harm to the company’s image caused by a mistake.

To be exact, when filling in the insurance policy for SMEs, the insurance brokerage firm omitted the existence of combustible materials used in the building of an industrial unit. Afterwards a fire broke out and completely destroyed the unit, which was then covered in the local news media. When the existence of these materials was made evident to the insurance company, they refused payment of compensation for loss. After several appeals against this decision, which prolonged the whole process, the client was finally paid through the public liability policy which legislation obliges all insurance brokers or insurance brokerage firms to have. However, this incident had an important impact in the area, causing harm to the insurance broker image.

Once the hidden quality cost to be quantified was determined, a first group of experts was selected, entirely made up of associated insurance mediators and adjusters. In accordance with Robbins (1994), the number of participants required for the decision making problems ranges between 5 and 7. For this reason 6 experts were selected for each of the expertise phases, providing them all with extensive information about the study to be made, the characteristics of the company, the business developed and the composition and importance of the client portfolio.

After analyzing the information, the first 6 experts made their first calculations of the cost of loss of image, whose limits ranged between 18,000 and 30,000€. Table 5 shows the valuations carried out by the experts just as they were facilitated through triplets of confidence, as well as the degree of confidence that the company has in each of these experts (we should remember that it is one of the two weighting factors to be considered).
Table 5. Cost of loss of image provided by the six experts, with indication of level of confidence assigned to each expert.

<table>
<thead>
<tr>
<th>Expert (i)</th>
<th>Cost of loss of image</th>
<th>Confidence in expert i (v_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20,000</td>
<td>22,000</td>
</tr>
<tr>
<td>2</td>
<td>22,000</td>
<td>24,000</td>
</tr>
<tr>
<td>3</td>
<td>21,000</td>
<td>23,000</td>
</tr>
<tr>
<td>4</td>
<td>18,000</td>
<td>22,000</td>
</tr>
<tr>
<td>5</td>
<td>19,000</td>
<td>23,000</td>
</tr>
<tr>
<td>6</td>
<td>25,000</td>
<td>27,000</td>
</tr>
</tbody>
</table>

At the same time, and as another of the weighting factors to be considered, a group of 5 experts was formed which included the manager of the insurance brokerage firm. Once suitably trained, their mission was to evaluate the position of the company on Crosby’s Maturity grid. Table 6 summarizes the experts’ opinions, the value of the membership function for each of the stages as defined by Crosby, and finally the Total stage index \( I_T \) obtained according to expression (8) and whose membership function is represented graphically in Figure 1.

Table 6. Results of the experts’ opinion about company’s membership to each of the phases of Crosby’s Maturity Grid.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Totally disagree</th>
<th>Disagree a lot</th>
<th>Slightly disagree</th>
<th>Slightly agree</th>
<th>Quite agree</th>
<th>Totally agree</th>
<th>( I_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>3</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td>A_2</td>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>1</td>
<td></td>
<td>0.80</td>
</tr>
<tr>
<td>A_3</td>
<td></td>
<td></td>
<td>3</td>
<td>2</td>
<td></td>
<td></td>
<td>0.48</td>
</tr>
<tr>
<td>A_4</td>
<td>4</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td>A_5</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
</tr>
</tbody>
</table>

\[ I_T \] 2.34
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Figure 1. Membership function of the total stage index

The application of the proposed methodology, taking $\beta = 0.5$, that is to say, by giving the same specific weight in the final valuation to each of the two weighting factors considered, the result is $\text{TFN}[20,292, 23,440, 26,106]$. That is to say, the cost of the loss of image ranges between 20,822 and 26,106 €, the maximum possible value being 23,440 €.

For the previous results to undergo Contra-Expertise, a new group of six independent experts was employed, made up of those responsible for the commercial and accident departments from different insurance companies. The proper treatment of the opinion expressed by the new experts (see Table 7, which gives both the valuations carried out by the new experts and the degree of confidence the company believes they deserve), following the proposed model and considering $\beta^* = 0.5$ again, it was finally possible to obtain a new TFN $[22,272, 22,967, 23,670]$. Consequently, the error made by the insurance brokerage firm meant a cost of loss of image ranging between 22,272 and 23,670 €, with the highest possible value being 22,967 €. If a point estimation is required, the previous TFN can be defuzzified. If it is done through the average value method, the loss of image experienced by the company rises to 22,970 €.
Table 7. Level of agreement of the new experts with values obtained after the first expertize, and indication of the level of confidence in each expert.

<table>
<thead>
<tr>
<th>Expert (i')</th>
<th>Valuations ( t_1^{i'} )</th>
<th>Valuations ( t_2^{i'} )</th>
<th>Valuations ( t_3^{i'} )</th>
<th>Confidence in expert ( i' ) (( u_1^{i'} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.250</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.220</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
<td>0.200</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.170</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.120</td>
</tr>
<tr>
<td>6</td>
<td>0.3</td>
<td>0.5</td>
<td>0.6</td>
<td>0.040</td>
</tr>
</tbody>
</table>

5. Conclusions

This article introduces fuzzy logic and the concept of possibility for use in quality management and measurement. To be exact, it proposes an innovating fuzzy model which will allow the estimation of any element a company considers as hidden quality cost.

The methodology proposed is sustained in the sequential process that develops from the opinions expressed by a group of experts about the cost to be quantified. This information is provided by experts by way of TFN in order to properly include the conditions of uncertainty and subjectivity that exist in the evaluations carried out. In order to aggregate and summarize the values given by the experts the use of BADD-UPOWA is proposed, establishing as weighting factors the level of confidence that the company has in each expert on the one hand, and the position the company occupies on Crosby’s famous Quality Administration Grid on the other hand, which is especially innovative and is carried out through an original tool designed for it.

Following this, in order to refine the results obtained, the model proposes their undergoing Contra-Expertise, thereby providing the analysis with greater consistency. Instead of using the traditional Contra-Expertise, the use of OWA-expertons was decided on, that is to say, expertons, where aggregation is made through OWAs by introducing similar weighting factors to those used in the first phase.

Once the theoretical model has been described, it has been applied to a case study: the quantification of the cost of loss of image in one insurance brokerage firm.

The proposed methodology makes it possible to aggregate subjectivities adequately, thereby reaching a certain level of objectivity and consequently this leads to a higher reliability of the final results. However, it does have a serious drawback which can limit its application in companies since it requires the participation of external experts whose opinions contribute to determining the value of hidden quality cost. This could be excessively expensive for organizations and also there would be the added difficulty of experts’ availability. With respect
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to this, we suggest that a future line of research should be to seek practical solutions which reduce costs and make it possible for a maximum number of organizations to access the model developed.

REFERENCES

Manuel E. Sansalvador, José M. Brotons


