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PRIORITIZED AGGREGATION FOR NON-HOMOGENEOUS GROUP DECISION MAKING IN WATER RESOURCE MANAGEMENT

Abstract. *This paper deals with non-homogeneous group decision making problems in water resource management, in which there exists a prioritization of decision makers. The group decision makers are partitioned into three sets: the officials from government, the experts in water resource management, the users of water resources. There exists a prioritization relationship over the different sets of decision makers. In order to aggregate a collective preference based on the aggregation of different individual preferences, we suggest that prioritization between decision makers can modeled by making the weights associated with a decision maker dependent upon the satisfaction of the higher priority decision maker. Then, a so-called prioritized weighted aggregation operator based on ordered weighted averaging (OWA) is utilized to aggregate the preference values provided by different decision makers. Finally, an application in water resource management is provided to illustrate the usefulness and how the prioritized aggregation works in practice.*

Keywords: *Prioritized aggregation; non-homogeneous group decision making; water resource management.*

JEL Classification D81, M12, M51

1. INTRODUCTION

The increasing complexity of the socio-economic environment makes it less and less possible for a single decision maker (DM) to consider all relevant aspects of a problem. As a result, many decision-making processes, in the real world, take place in group settings (Kim et al., 1999; Merigó et al., 2012; Xu and Da, 2010; Xu et al., 2010; Xu et al., 2013; Xu et al., 2012; Xu and Wang, 2011; Xu and Wang, 2012a; Xu and Wang, 2012b; Xu, 2006; Zeng and Su, 2012). A group decision making (GDM) problem may be defined as a decision situation in which (1) there are two or more experts, each of them characterized by his/her own perceptions, attitudes, motivations, and personalizes, (2) who recognize the existence of a common problem, and (3) who attempt to reach a collective decision (Delgado et al., 1998; Herrera et al., 1997; Herrera et al., 1998). Sometimes, one may admit that the various decision makers that give the options are not equally important. For example, in order to decide whether a paper would be published in a journal or not, the journal manager first sends a paper to the reviewers, then the reviewers begin to review the paper, after all the reviewers return their suggestions to the editorial office, then the journal editorial committee would decide whether the paper would be published or not according to the reviewers' comments. Actually, we can take this process as the group decision making, there exists two subgroups in this GDM problem: the reviewers group and editorial committee. The reviewers only have the privilege to give comments or suggestions, the final decision is made by the editorial committee. The editorial committee could accept or reject the reviewers' suggestions. It is clearly that the weight of editorial committee is more important than the reviewers, and thus should be assigned a larger weight. Another example is discussed in this paper for water resource management in China. The water allocation problem is also a group decision making problem, it involves the government (central government, hydrographic basin committees and the local government), experts and users to participate in the discussion the alternatives. In China, the government has the absolute power to make the decision, the experts from some aspects could provide their suggestions to make the decision more scientific according to their respective professional angle, and the users only could provide their requirements and comments. Thus, for this group decision making problem, the government has the largest weights, and the experts would be the second, and the users have the smallest weights. In this kind of group decision making, there exists priorities between decision makers. We call it non-homogeneous prioritized group decision making (PGDM).

Prioritized multi-criteria decision making problem was first addressed by Yager (Yager, 2004), in which there exists a prioritization of criteria. Afterwards, Yager proposed the prioritized aggregation operators, such as prioritized "anding" and "oring" operator (Yager, 2008), prioritized OWA operator (Yager, 2009). Amin and Sadeghi (2010) used the prioritized aggregation operators to aggregate a preference voting problem. Yan et al. (Yan et al., 2011) propose a prioritized weighed aggregation operator based on ordered weighted averaging (OWA) operator(Merigó and Gil-Lafuente, 2009; Merigó and Gil-Lafuente, 2011; Merigó et al., 2013; Torra, 1997;

Yager, 1988; Yager, 1993) and triangular norms (t-norms). However, all the existed research are only paid attention on the prioritized criteria, there is no research focus on the prioritized individuals, as it is also a common problem in real decision making. This is the focus of this paper. In this paper, we use the prioritized OWA operator to solve the non-homogenous group decision making problem in which there exists prioritization of decision makers. The rest of this paper is structured as follows. Section 2 proposes the prioritized OWA operator for non-homogeneous group decision making problems, and study some of its desired properties. Section 3 gives an case study for water resource management in China. Section 4 concludes the paper.

2. PRIORITIZED AGGREGATION OPERATOR

In the group decision making, some of the decision makers usually are regarded as prior to others, such as the officers of government, the leaders in a company, etc. In this case, some decision makers shall be considered as a matter of priority. In order to make a proper decision in this kind of group decision making problems, we can first construct the prioritization relations among the decision makers, and then calculate the overall scores of each alternative by using the prioritized aggregation operators (Yager, 2008; Yager, 2009). In what follows, we develop an operator called prioritized OWA operator for non-homogeneous group decision making problem.

We assume that all the decision makers $D=\{D_1,D_2,...,D_n\}$ can be portioned into q distinct categories $H_1,H_2,...,H_q$ such that $H_i=\{D_{i1},D_{i2},...,D_{in_i}\}$, here D_{ij} are the decision maker in category H_i, and there exists a prioritization among the categories $H_1 \succ H_2 \succ ... \succ H_q$ where symbol "$\succ$" denotes "prior to". The total set of decision makers is $D=\bigcup_{i=1}^{q} H_i$, and $\sum_{i=1}^{q} n_i = n$. Also, suppose $X=\{x_1,x_2,...,x_m\}$ indicates the set of alternatives. We assume that, for any alternative x in X, we have for each decision maker D_{ij} a value $D_{ij}(x)\in[0,1]$, indicating its satisfaction to decision maker D_{ij}. Our aim is to rank the alternatives in X. In Fig. 1 we show the positioning of the decision maker. This priority hierarchy into two cases: (1) **Strict priority order**, if each priority level has only one decision maker this type, i.e., $N_k=1$ for $k=1,2,...,q$. (2) Otherwise the priority order is called **weakly ordered prioritization**.

$D_{11}, D_{12}, ..., D_{1n_1}$
$D_{21}, D_{22}, ..., D_{2n_2}$
$\vdots$
$\vdots$
$D_{q1}, D_{q2}, ..., D_{qn_q}$

Figure 1. Prioritization of decision maker

Yager(2008) introduced a prioritized scoring (PS) operator $F:[0,1]^n \to [0,1]$ such that $F((a_{11},...,a_{1n_1}),...,(a_{q1},...,a_{qn_q})) = \sum_{i=1}^{q} \sum_{j=1}^{n_i} w_{ij} a_{ij}$. Using this aggregation operator, we can calculate $D(x)$ for alternative x as

$$D(x) = F(D_{ij}(x)) = \sum_{i=1}^{q}\left(\sum_{j=1}^{n_i} w_{ij} D_{ij}(x)\right) \tag{1}$$

Here the weights w_{ij} are a function of x and will be used to reflect the priority relationship. Yager (Yager, 2008) proposed some methods to obtain the weights for a given alternative x. In the following, we shall utilize the OWA operator to aggregate the priority category $H_i = \{D_{i1}, D_{i2}, ..., D_{in_i}\}$. OWA operator is similar to a weighted mean, but with the values of the variables previously ordered in a decreasing way. The weights of OWA is associated with a particular ordered position of the arguments, which contrary to the weighted means. The OWA has some interesting properties such as monotonicity, idempotency, boundary. In this paper, we use the OWA to obtain satisfaction degree for each priority level and suppose

$$Sat_i = \mathrm{OWA}\omega_i(D_{i1}(x), D_{i2}(x), ..., D_{in_i}(x)) = \sum_{k=1}^{n_i} \omega_{ik} b_{ik}(x) \tag{2}$$

where ω_i is the OWA weighting vector associated with each priority category H_i and $b_{ik}(x)$ is the k th largest of $D_{ij}(x)$. The components ω_{ik} of ω_i are such as $\omega_{ik} \in [0,1]$, and $\sum_{k=1}^{n_i} \omega_{ik} = 1$. There are a class of methods to determine the associated weights ω_{ik} of ω_i, such as linguistic quantifiers (Yager, 1988), orness measure (Yager, 1988), dispersion measure (Yager, 1988), O'Hagan's maximum entropy measure(O'Hagan, 1988), normal distribution based method (Xu, 2005), etc.

To model the priority relationship, Yager suggested that the lower priority will become important with the higher degree of higher priority level, i.e., the priority weights are dependent upon the satisfaction of higher priority level. According to this idea, our first step is to obtain the priority induced importance weights of each priority level H_i a priority weight T_i. In particular, for priority level H_1, we have $T_1 = 1$. For priority level H_2, $T_2 = T_1 Sat_1$. For priority level H_3, we have $T_3 = T_1 T_2 Sat_2$. More succinctly and generally, we can induce the priority weights for priority level H_i as

$$T_i = \prod_{k=1}^{i} Sat_{k-1} \quad (3)$$

with the understanding that $Sat_0 = 1$.

We now see that for the priority level H_i, we have a priority weight T_i and its satisfaction Sat_i. In this way, we can get an aggregated value for each alternative by

$$D(x) = \sum_{i=1}^{q} T_i Sat_i \quad (4)$$

We call it prioritized OWA (POWA) operator.

We can use the above Eqs.(2)-(4) to obtain the over satisfaction for each alternative. Note that for the all the priority level H_i ($i = 1, 2, ..., q$), the sum of all the level do not meet the normalization condition, i.e., $\sum_{i=1}^{q} T_i \neq 1$. At the same time, we also can get the normalized priority based importance weight u_i associated with category H_i ($i = 1, 2, ..., q$).

$$u_i = \frac{T_i}{\sum_{j=1}^{q} T_j} \quad (5)$$

Then, we can obtain the overall score of each alternative

$$D(x) = \sum_{i=1}^{q} u_i Sat_i \quad (6)$$

Note that if $Sat_k = 0$ in Eq.(3), then $T_i = 0$ for all $i > k$, from this we see that in the case of $S_k = 0$, we have $u_i = 0$. Especially, if $Sat_1 = 0$ then $u_i = 0$ for $i > 1$ and hence $u_1 = 1$.

Remark 1. (1) If $n_i = 1$ for all i, then by Eq.(2), we have $Sat_i = D_i(x)$, and POWA operator is reduced to the PA operator proposed by Yager (Yager, 2009).

Theorem 1(Monotonicity). Let D be the POWA operator defined by Eq. (4). If $D_{ij}(x) > D_{ij}^{'}(x)$, for all $i = 1, 2, ..., q$, $j = 1, ..., n_i$, then

$$D((D_{11}(x), ..., D_{1n_1}(x)), ..., (D_{q1}(x), ..., D_{qn_q}(x))) >$$

$$D((D'_{11}(x),...,D'_{1n_1}(x)),...,(D'_{q1}(x),...,D'_{qn_q}(x))) \quad (7)$$

Proof. Since $D_{ij}(x) > D'_{ij}(x)$, for all $i=1,2,...,q$, $j=1,...,n_i$, by Eq.(2), we have

$$Sat_i > Sat'_i$$

By Eq.(3), $T_i > T'_i$. Then, By Eq.(4), we have

$$D((D_{11}(x),...,D_{1n_1}(x)),...,(D_{q1}(x),...,D_{qn_q}(x))) >$$
$$D((D'_{11}(x),...,D'_{1n_1}(x)),...,(D'_{q1}(x),...,D'_{qn_q}(x))).$$

Theorem 2 (Commutativity). Let D be the POWA operator defined by Eq. (4). Then

$$D((D_{11}(x),...,D_{1n_1}(x)),...,(D_{q1}(x),...,D_{qn_q}(x))) =$$
$$D((D'_{11}(x),...,D'_{1n_1}(x)),...,(D'_{q1}(x),...,D'_{qn_q}(x))) \quad (8)$$

where $(D'_{i1}(x), D'_{i2}(x),...,D'_{in_i}(x))$ is any permutation of the arguments $(D_{i1}(x), D_{i2}(x), ..., D_{in_i}(x))$ for all $i=1,2,...,q$.

Proof. Since $(D'_{i1}(x), D'_{i2}(x),...,D'_{in_i}(x))$ is any permutation of the arguments $(D_{i1}(x), D_{i2}(x),...,D_{in_i}(x))$ for all $i=1,2,...,q$, by Eq.(2), we have

$$Sat_i = Sat'_i$$

and by Eqs.(3) and (4), we have

$$D((D_{11}(x),...,D_{1n_1}(x)),...,(D_{q1}(x),...,D_{qn_q}(x))) >$$
$$D((D'_{11}(x),...,D'_{1n_1}(x)),...,(D'_{q1}(x),...,D'_{qn_q}(x)))$$

Theorem 3 (Boundary). Let D be the POWA operator defined by Eq. (4). Then

$$\min\{D_{ij}(x)\} < D((D_{11}(x),...,D_{1n_1}(x)),...,(D_{q1}(x),...,D_{qn_q}(x))) < \max\{D_{ij}(x)\} \quad (9)$$

Proof. From Eq.(2) for calculating Sat_i, we can obtain that

$$\min\{D_{ij}(x)\} < Sat_i < \max\{D_{ij}(x)\}$$

Since $D_{ij} \in [0,1]$, then by Eq.(3), we have $T_i \in [0,1]$, and thus

$$\min\{D_{ij}(x)\} < D((D_{11}(x),...,D_{1n_1}(x)),...,(D_{q1}(x),...,D_{qn_q}(x))) < \max\{D_{ij}(x)\}$$

3. CASE STUDY

In this section, a description is given of a real world application of a non-homogeneous group decision making problem to support the choice of an alternative for water resource allocation in Zhanghe, a river located in Shangxi province, China (Wang and Tong, 2011). Zhanghe is an important river of Haihe hydrographic basin. It originates from churchyard of Shanxi province, pass through the boundary of Hebei, Henan two provinces. In recent years, due to the neighbor provinces compete to develop, the use of water are increased rapidly, and this causes the yield of water decreased, and then causes the water disputes. In order to solve the disputes, there must come on the water allocation scheme for these areas.

First of all, it is important to note that, the water resource allocation problem is a complex problem, because there are so many participants in this problem. We can divide all these participants into three categories: (1) the government (H_1), (2) experts from water resources (H_2) and (3) users (H_3). In China, the government is the centers of decisions on water resources management. Furthermore, the government includes central government (D_{11}), hydrographic basin committees (D_{12}) and the local government (D_{13}). In China, the central government is the highest management institution, and its representative is Ministry of Water Resources. Ministry of Water Resources do the policy making to assure the reasonable use of water resources. Hydrographic basin committees are the actually managers of water resources, and the main organizers of water allocation. The local government proposes the original water resources requests according to local economic development, hydrology condition and environment etc. The experts from water resources have the knowledge of one aspect, such as environment protection (D_{21}), hydrology science (D_{22}), law (D_{23}), etc. These experts can analysis the regulation of the users, and construct a series of mathematical models to derive the alternatives sets for the water resources allocation, and assure the allocation more scientific. The users include the industry users (D_{31}), agriculture users (D_{32}) and resident users (D_{33}).

Now, there are above three categories to make the water resource allocation decision. In China, the government (H_1) is the centers of the decisions on water resources management, and hence, the representatives of government have the highest weights. The experts have one of the aspect knowledge and can make the allocation reasonable, and thus, the importance is lower than the government, but higher than the users. The users are from different aspects. They can propose their requests and obey the allocation scheme, and have the lowest importance. Therefore, the water resource allocation problem is a non-homogeneous group decision making problem, and also the categories are prioritized such that $H_1 \succ H_2 \succ H_3$. Assume that there are five water resources allocation scheme (alternatives) x_i ($i = 1, 2, ..., 5$), each categories of decision makers appoint one representative from different area to participate the meeting to give their respective satisfaction to each alternative, here, there are 9 decision makers, 3 from different level governments (D_{11}, D_{12}, D_{13}), 3 experts from different areas (D_{21}, D_{22}, D_{23}), and 3 user representatives from different aspects (D_{31}, D_{32}, D_{33}). The 9 decision makers give their scores $D_{ij}(x)$ according to each alternative x_l ($l=1,2,...,5$) as shown in Table 1.

Table 1. Satisfaction degree of each decision maker regarding each alternative

	D_{11}	D_{12}	D_{13}	D_{21}	D_{22}	D_{23}	D_{31}	D_{32}	D_{33}
x_1	0.7	0.8	0.8	0.7	0.8	0.7	0.9	0.6	0.7
x_2	0.9	0.6	0.8	0.5	0.7	0.8	0.6	0.5	0.8
x_3	0.8	0.9	0.7	0.8	0.9	0.7	0.8	0.7	0.7
x_4	0.5	0.7	0.9	0.9	0.5	0.5	0.5	0.9	0.6
x_5	0.6	0.8	0.5	0.8	0.6	0.6	0.7	0.8	0.9

In order to determine the best alternative, we first need to determine the weights associated with OWA Eq.(2). A very useful approach to obtain the weights is the functional method introduced by Yager using linguistic quantifiers, such as “Most”, “At least half” and “Average”. Any linguistic quantifier Q can be expressed as a fuzzy set, such that $Q(0)=0$, $Q(1)=1$, and $Q(x)>Q(y)$ for $x>y$. Using the linguistic quantifier Q, the weight can be obtained as follows:

$$\omega_j = Q\left(\frac{j}{n}\right) - Q\left(\frac{j-1}{n}\right),\ j=1,2,...,n \tag{10}$$

Zadeh (Zadeh, 1983) defined Q as follows:

$$Q(r)=\begin{cases}0, & \text{if } r<a \\ \dfrac{r-a}{b-a}, & \text{if } a\le r\le b \\ 1, & \text{if } r>b\end{cases} \tag{11}$$

Now, we use the POWA operator to aggregate as follows:

(1) We first calculate the degree of satisfaction for each priority level via OWA operator as follows (assume we use the linguistic quantifier “Most”, by Eqs.(10) and (11), the weight vector is $\omega_i=(0.066,0.666,0.268)^T$, ($i=1,2,3$)):

$$Sat_1(x_1)=OWA_{\omega_1}(D_{11}(x_1),D_{12}(x_1),D_{13}(x_1))=OWA_{\omega_1}(0.7,0.8,0.8)=0.7732\ ,$$

$$Sat_2(x_1)=OWA_{\omega_2}(D_{21}(x_1),D_{22}(x_1),D_{23}(x_1))=OWA_{\omega_2}(0.7,0.8,0.7)=0.7066\ ,$$

$$Sat_3(x_1)=OWA_{\omega_3}(D_{31}(x_1),D_{32}(x_1),D_{33}(x_1))=OWA_{\omega_3}(0.9,0.6,0.7)=0.6864\ .$$

(2) Then, according to Eq.(3), we calculate the priority weight for each priority level as follows:

$$T_1=1,\ T_2=Sat_0\cdot Sat_1=0.7732\ ,\ T_3=T_2\cdot Sat_2=0.5463$$

(3) According to Eq.(4), we obtain the global prioritized aggregated value as follows:

$$D(x_1)=\sum_{i=1}^{3}T_i Sat_i = 1\cdot 0.7732+0.7732\cdot 0.7066+0.5463\cdot 0.6864=1.6945$$

Similarly, we have

$$D(x_2)=1.5330\,,\ D(x_3)=1.8176\,,\ D(x_4)=1.2127\,,\ D(x_5)=1.2264$$

Thus, the ranking order of prioritized aggregation values are as:

$$D(x_3)\succ D(x_1)\succ D(x_2)\succ D(x_5)\succ D(x_4)$$

Thus, the best alternative is x_3.

4. CONCLUSIONS

In this paper, we have concerned the non-homogeneous group decision making problems in which there exists a prioritization of decision makers. Based on the prioritized scoring (PS) operator (Yager, 2008) which is firstly proposed to solve the prioritization problem between criteria, this paper proposed prioritized OWA operator to deal with the non-homogeneous group decision making problems, and study some of its desired properties. The priority weights with the lower priority are related to the satisfactions of the higher priority group. Finally, an application in water resource management in China is provided to illustrate the usefulness and how the prioritized aggregation works in practice.

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